ON THE NUMBER OF SPIRAL NEBULÆ.

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The probable total number of the spiral nebulæ is a matter of considerable importance in all theories bearing on the constitution of these objects and their place in the sidereal plan. Prior to the introduction of photographic methods fewer than ten thousand nebulæ were known. One of the first results deduced by Director Keeler from the program of nebular photography, which he inaugurated with the Crossley Reflector, was that there exist many thousand very small, uncatalogued nebulæ, the great majority of which are undoubtedly spirals. Early in the course of this program, and before photographs of many regions were available, he estimated that there were at least 120,000 small spirals, and regarded this estimate as a very conservative one.1 On completing the original Keeler program Perrine² came to the conclusion, from counts of small nebulæ made on fifty-seven of the one hundred and four regions of that program, that at least 500,000 small nebulæ were within reach of the Crossley reflector, and deemed it not improbable that the total might ultimately exceed 1,000,000. Later, Fath,3 working with the 60-inch Reflector at Mt. Wilson, took a series of 130 plates at the centers of the Kapteyn areas, on which 1,031 nebulæ were recorded, and estimated that the number within reach of the 60-inch Reflector with exposures of one hour on Lumière . Sigma plates (an approximate equivalent to the exposures of the Crossley program) was 162,000.

The great numbers of small spirals found on nearly all plates of regions distant from the Milky Way long since led me to the belief that Perrine's estimate of half a million was likely to be under,

¹ Ap. Jour., 11, 325, 1900, and Publ. Lick Obs., 8. ² Lick Obs. Bull., 3, 47, 1904.

⁸ Astr. Jour., 28, 75, 1913.

rather than in excess of, the truth. The extent of the existing photographic material now makes possible and useful a new determination of the number of the spiral nebulæ. Descriptions of 762 nebulæ and clusters are being published in a University of California semi-centennial volume, and, in preparing this paper, I have made counts of the numbers of small nebulæ occurring on all the available regions of the complete Crossley program covered by the above list, extending from 1898; when systematic work was commenced at Mt. Hamilton with this instrument, to February 1, 1918.

In all, 439 regions were available for these counts, giving a total of 5,698 small, uncatalogued nebulæ. To these must be added the 513 spirals described in the list, making the total number of all nebulæ found in all the regions 6,211. In the belief that practically all these small nebulæ are spirals, I have designedly omitted the diffuse nebulosities and the planetaries.

The exposed area of a Crossley plate is about nine tenths of a square degree; the edges and corners of this area are, however, so poorly defined, owing to the distance from the optical axis, that only the brighter of the small nebulæ can be picked up in these outer parts of the plate. The greater proportion of the small nebulæ are found in the more central parts having an area of six tenths of a square degree, or less. I have assumed 0.75 of a square degree as the average effective area on which the counts were made; this is certainly somewhat in excess, but is an error on the conservative side.

We shall first assume that these 439 regions are sufficiently uniform in their distribution to be taken as a fair representation of the whole sky; their area will be 329.25 square degrees. If the proportion shown by these regions holds over the entire sky, we should expect the number of the spiral nebulæ to be 778,000. A plot of the regions shows that their distribution may well be regarded as an approximately uniform one. There is, it is true, a marked concentration of regions in the vicinity of the north galactic pole, but this would appear to be balanced by a similar preponderance of regions in the Milky Way between 17 and 20 hours of right ascension (see Fig. 1). If we divide the celestial sphere into two equal areas, one consisting of a zone 60° wide extending for 30° on each side of the galactic plane, and the other comprising the two zones of 60° radius

about the two galactic poles, we shall find that 217 regions, or 49.4 per cent., are located in the first, or galactic area, and 222 regions, or 50.6 per cent., in the polar areas. The balance in the distribution of the regions in the galactic and extra-galactic zones is, then, nearly perfect.

On the other hand, bearing in mind the well-known concentration of the spiral nebulæ in the vicinity of the north galactic pole, and



FIG. I. Chart showing distribution of regions on which small nebulæ were counted.

the fact that the present program deals primarily with the regions containing spirals, it is necessary to investigate further any possible effect which a concentration of regions near the north galactic pole might have upon the resulting estimate. This objection was urged by Fath (loc. cit.) in explanation of the difference between his estimate of 162,000 and the 500,000 of Perrine. He found, on plotting Perrine's regions, that 33 per cent. of these were within 45° of the north galactic pole, while less than 20 per cent. of the Kapteyn areas were within this distance. In the present program I find that 117 regions, or 26 per cent. of the regions which I have used, are situated within 45° of the north galactic pole, in an area amounting to but 14.6 per cent. of the sky, and these 117 regions contain 2,997 spirals, or about 48 per cent. of the whole.

To avoid the effect of this concentration in the north polar galactic area, it will be advisable to subdivide the material available. We

may divide the celestial sphere into four areas, two of which will be the zones of 45° radius about the two galactic poles; a third area will comprise the two zones, each 15° in width, extending from -30° to -45° and from $+30^{\circ}$ to $+45^{\circ}$ galactic latitude, and the fourth will be the zone 60° wide extending from -30° to $+30^{\circ}$ galactic latitude. The results are indicated in the following short table:

Galactic Latitude.	No.	Sq.	No.	No. per	No. in
	Regions.	Deg.	Spirals.	Sq. Deg.	Area,
$\begin{array}{c} + 45^{\circ} \text{ to } + 90^{\circ} \dots \\ - 45^{\circ} \text{ to } - 90^{\circ} \dots \\ \pm 30^{\circ} \text{ to } \pm 45^{\circ} \dots \\ - 30^{\circ} \text{ to } + 30^{\circ} \dots \end{array}$	117	88.50	2,997	34	205,000
	43	32.25	918	28	169,000
	62	46.50	1.117	24	204,000
	217	162.75	1,179	7	144,000
Totals	439		6,211		722,000

It will be seen from the above that there is a concentration of the smaller nebulæ in the vicinity of the north galactic pole similar to that which obtains among the larger, visually discovered, spirals, and that the density about the south galactic pole is somewhat less marked. The data given in the third line of the table are of special interest, showing that the small spirals persist to a distance of at least 60° from the galactic poles, with only a slight diminution in the degree of density which obtains in the polar areas.

As this revised estimate, 722,000, is equal to that of Fath plus that of Perrine, with several thousand to spare, a discussion of the possible reasons for the discrepancy becomes imperative. The following points may be considered:

A. It may be urged that my count has not been sufficiently conservative, and that I have possibly included many spurious objects. The detection of the faintest and smallest nebulæ is very largely a matter of experience, and all who have worked with photographic plates soon learn, by hard necessity, to recognize the average flaw at a glance. A very large proportion of the objects are unmistakably nebulæ. As to the faintest nebulæ, it is astonishing how faint and small are the nebulæ which two "clean" duplicate plates will reproduce. For a large proportion of my regions no duplicate plates exist, and I have been necessarily guided by the experience derived

from regions taken in duplicate. I am unwilling at present to admit that as many as five per cent. of the nebulæ counted by me are spurious. Even if twenty per cent. were spurious, we should still have to account for over half a million nebulæ.

B. The theory may be advanced that the small spirals occur in greatest profusion in the regions contiguous to the larger members of the class, which might explain why fewer nebulæ were found by Fath. His plates were taken at the centers of the Kapteyn areas where the larger nebulæ would be included only by chance, whereas, from the purpose of the Crossley nebular program, nearly all the plates have some N. G. C. object central. This point is difficult to prove or disprove without a special investigation comprising many plates taken at random in the galactic north polar region. It is certain that the small spirals frequently show a gregarious tendency; sometimes one half of a plate will record many small spirals while the other half records very few. The greatest number of nebulæ found on a single plate was 304 (checked by a duplicate plate) in the region of 12^h 55^m, + 28° 30' (shown in Fig. 2); the region about N. G. C. 4826, less than 7° distant from this, shows but 2. While the small nebulæ are evidently quite irregular in their distribution, it would seem that the large number of regions included in this discussion is sufficient to afford a true representation of their average frequency.

C. Sharp focus and perfect images are essential for the detection of the smallest and faintest spirals. On plates where large numbers of small nebulæ are found, the majority are, as a rule, detected in the area 20' in radius about the optical axis as center, comprising only 0.35 of a square degree. At a distance of 30' from the optical axis the parabolic images are very poor, and only the brighter of the small nebulæ can be detected in these regions, the faintest nebulæ being obliterated by the blurring and spreading of the image. Dr. Fath used very large plates, $6\frac{1}{2} \times 8\frac{1}{2}$ inches in size, in his work on the number of the small nebulæ, with the 60-inch reflector, and it appears that he used almost this full area, inasmuch as he states that the used area of his plates was 1.88 square degrees. As the full area of the Crossley plate is 0.9 square degrees (and the outer portions of this are so poor because of the parabolic distortions that

the effective area used in my counts is believed to be less than 0.75 square degrees) this would mean, if the two reflectors were of the same focal length, that the images on one half of the angular area of the large plates used by Fath were worse than those in the rejected edge strips of the Crossley plates. But the greater focal



FIG. 2. Region of many small nebulæ at $\cdot 12^{h}$ 55^m + 28° 30'. 249 small nebulæ in an area $38' \times 39'$.

ratio of the 60-inch would increase this disadvantage, even allowing for certain advantages which might partially counterbalance this due to the greater linear scale of the plates. The following com-

parisons of the aberrations of the image of an infinitely distant object in the focal plane of a single, perfect, parabolic mirror, as calculated from Schwarzschild's formulæ⁴ for the Crossley and the 60-inch reflectors, will illustrate this point.

	THE CROSSLEY REFLECTOR;	Focal Ratio = $1:5.8$.	
30'	Dist, from Optical Axis.	D.stortion by Field Curvature. 2".8	Distortion by Coma. IO".I
41'	(corner of plate)	5″.2	13″.7
	THE 60-INCH REFLECTOR;	Focal Ratio = 1 : 5.0.	
30' 61'	(corner of plate)	····· 3″·3 ···· 13″.5	13″.5 27″.5

From my own experience in counting these minute objects on the Crossley plates it would appear to me that the actual *effective* area used by Fath must have been very much less than the 1.88 square degrees assumed by him in his calculations.⁵ It is my opinion also that the Lumière Sigma plates which he used are not the best for the end in view. These plates are of very great speed, and are invaluable for some purposes. I have long since ceased to use them for nebular work, however, believing that the slightly slower, but beautifully "clean" Seed 27 and Seed 23 plates really show the faintest details better. With the smaller grain and clear background existing in the Seed plates, very small and faint nebulæ "stand out" much more plainly than on the more rapid Sigma plates.

D. It is not impossible that a considerable proportion of the thirty or so plates which Fath took within 45° of the north galactic pole happened to strike regions of comparatively few small nebulæ. Had he chanced to include four such regions as the following:

⁴ Untersuchungen zur geometrischen Optik," Abh. Kön. Ges. d. Wiss. zu Göttingen, Math.-Phys. Kl., N. F., 4, 1, 2, and 3, 1905.

⁵ Since the completion of the manuscript of this paper Dr. Fath has published a note on "The Probable Number of Nebulæ" in A. J., 728, March 12, 1918, in answer to a letter in which Dr. Perrine had called his attention to the large angular area and the increased parabolic distortions in the outer parts of the plates as a factor in the smaller estimate made by Dr. Fath. Dr. Fath finds, by taking the counts found from the areas 40' square in the center of the Mt. Wilson plates, that there is a marked increase in the number of nebulæ found, amounting to about 60 per cent., and increasing his earlier estimate to 262,000.

a	i	5.			No. of S	Small Nebula
11 ^h 9 ^m	+ 55°	34'	 	 		107
II ^h 22 ^m	+ 17°	46'	 	 		69
12 ^h 15 ^m	+ 6°	1'	 	 		81
12 ^h 55 ^m	+ 28°	30'	 	 		304
						561

his estimate would have been increased by at least 60 per cent. (he found 864 new nebulæ in all, and the larger plates he used would have added a number to those counted on the Crossley plates).

Perhaps all the reasons outlined above may be regarded as contributing to an explanation of the difference between Fath's estimate of 162,000, and the larger ones due to Perrine and the present investigation. Of these, the possibility noted under (B), that the small nebulæ may conceivably occur in greatest profusion contiguous to the visually discovered objects of the N. G. C., would appear to be the only reason for changing the larger estimates, and this evidence is only to be secured by taking many additional plates at random.

In conclusion, I see no reason, at present existing, for changing the estimate made in this paper, that at least 700,000 small spirals are within reach of large reflecting telescopes. Because of the fact that the faintest and smallest members of the class are, in general, discernible only in the more central regions of the plate, I am inclined to regard the figure given above as, if anything, an underestimate, and consider it very probable that the total number accessible may well exceed one million.